

## **Design of a Model Reference Adaptive Controller Using Modified MIT Rule for a Second Order System**

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### **Abstract**

Sometimes conventional feedback controllers may not perform well online because of the variation in process dynamics due to nonlinear actuators, changes in environmental conditions and variation in the character of the disturbances. To overcome the above problem, this paper deals with the designing of a controller for a second order system with Model Reference Adaptive Control (MRAC) scheme using the MIT rule for adaptive mechanism. In this rule, a cost function is defined as a function of error between the outputs of the plant and the reference model, and controller parameters are adjusted in such a way so that this cost function is minimized. The designed controller gives satisfactory results, but is very sensitive to the changes in the amplitude of reference signal. It follows from the simulation work carried out in this paper that adaptive system becomes unstable if the value of adaptation gain or the amplitude of reference signal is sufficiently large. This paper also deals with the use of MIT rule along with the normalized algorithm to handle the variations in the reference signal, and this adaptation law is referred as modified MIT rule. The performances of the proposed control algorithms are evaluated and shown by means of simulation on MATLAB and Simulink.

**Keywords:** Model Reference Adaptive Control, Adaptive Controller, MIT rule, Normalized Algorithm, Modified MIT rule.

## 1. Introduction

A control system is a device that regulates or controls the dynamics of any other plant or process. Adaptive control is one of the widely used control strategies to design advanced control systems for better performance and accuracy. Model Reference Adaptive Control (MRAC) is a direct adaptive strategy with some adjustable controller parameters and an adjusting mechanism to adjust them. As compared to the well-known and simple structured fixed gain PID controllers, adaptive controllers are very effective to handle the unknown parameter variations and environmental changes. An adaptive controller consists of two loops, an outer loop or normal feedback loop and an inner loop or parameter adjustment loop. This paper deals with designing of adaptive controller with MRAC scheme using MIT rule to control a second order system.

## 2. Model Reference Adaptive Control

### 2.1 Principle of working

Model Reference Adaptive Control strategy is used to design the adaptive controller that works on the principle of adjusting the controller parameters so that the output of the actual plant tracks the output of a reference model having the same reference input.

### 2.2 Components

Reference Model: It is used to give an idyllic response of the adaptive control system to the reference input.

Controller: It is usually described by a set of adjustable parameters. In this paper only one parameter  $\theta$  is used to describe the control law. The value of  $\theta$  is primarily dependent on adaptation gain.

Adjustment Mechanism: This component is used to alter the parameters of the controller so that actual plant could track the reference model. Mathematical approaches like MIT rule, Lyapunov theory and theory of augmented error can be used to develop the adjusting mechanism. In this paper we are using MIT rule with Normalized Algorithm and the technique is then referred as Modified MIT rule.

The basic block diagram of MRAC system is shown in the fig.1. As shown in the figure,  $y_m(t)$  is the output of the reference model and  $y(t)$  is the output of the actual plant and difference between them is denoted by  $e(t)$ .

$$e(t) = y(t) - y_m(t) \quad (1)$$

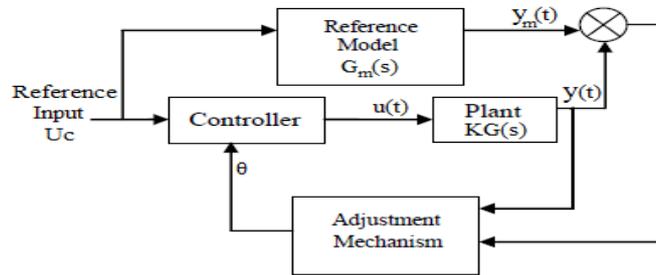


Figure 1: Model Reference Adaptive Control System.

### 3. MIT Rule

MIT rule was first developed in 1960 by the researchers of Massachusetts Institute of Technology (MIT) and used to design the autopilot system for aircrafts. MIT rule can be used to design a controller with MRAC scheme for any system.

In this rule, a cost function is defined as,

$$J(\theta) = e^2/2 \tag{2}$$

Where  $e$  is the error between the outputs of plant and the model, and  $\theta$  is the adjustable parameter.

Parameter  $\theta$  is adjusted in such a fashion so that the cost function can be minimized to zero. For this reason, the change in the parameter  $\theta$  is kept in the direction of the negative gradient of  $J$ , that is

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} \tag{3}$$

From Eq. (2),

$$\frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta} \tag{4}$$

Where, the partial derivative term  $\frac{\partial e}{\partial \theta}$  is called as the sensitivity derivative of the system. This term indicates how the error is changing with respect to the parameter  $\theta$ . And eq. (3) describes the change in the parameter  $\theta$  with respect to time so that the cost function  $J(\theta)$  can be reduced to zero. Here  $\gamma$  is a positive quantity which indicates the adaptation gain of the controller.

Let us assume that the process is linear with transfer function  $KG(s)$ , where  $K$  is an unknown parameter and  $G(s)$  is a second order known transfer function. Our goal is to design a controller so that our process could track the reference model with transfer function  $G_m(s) = K_oG(s)$ , where  $K_o$  is a known parameter.

From Eq. (1),

$$E(s) = KG(s)U(s) - K_o G(s)U_c(s) \quad (5)$$

Defining a control law,

$$u(t) = \theta * u_c \quad (6)$$

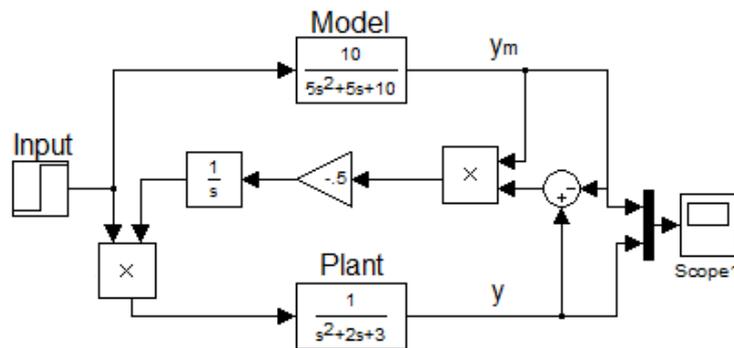
From Eqs. (5-6), and taking partial differentiation,

$$\frac{\partial E(s)}{\partial \theta} = KG(s)U_c(s) = \frac{K}{K_o} Y_m(s) \quad (7)$$

From Eq.(4) and Eq.(7), we will get,

$$\frac{d\theta}{dt} = -\gamma e \frac{K}{K_o} y_m = -\gamma' e y_m \quad (8)$$

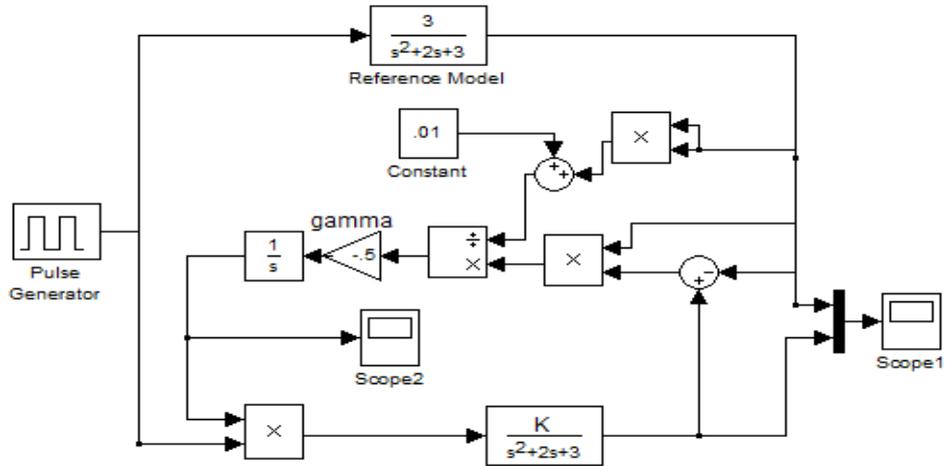
Eq.(8) will give us the law for adjusting the parameter  $\theta$  and the Simulink model is shown in fig.(2). It has been seen from simulation results that the response of the plant depends upon the adaptation gain  $\gamma'$ . In some industrial plants, larger values of  $\gamma'$  can cause the instability of the system and selection of this parameter is very critical.



**Figure 2:** Simulink diagram of Model Reference Adaptive Controller with MIT rule.

#### 4. Normalized Algorithm

The designed controller using MIT rule gives satisfactory results but is very sensitive to the changes in the amplitude of the reference input. For large values of reference input, system may become unstable. Hence to overcome this problem, Normalized algorithm is used with MIT rule to develop the control law.



**Figure 3:** Simulink diagram of Model Reference Adaptive Controller with modified MIT rule

.Normalized algorithm modifies the adaptation law in the following manner,

$$\frac{d\theta}{dt} = \frac{-\gamma e \varphi}{\alpha + \varphi' \varphi} \tag{9}$$

Where,  $\varphi = \frac{\partial e}{\partial \theta}$  and  $\alpha$  ( $\alpha > 0$ ) is introduced to remove the difficulty of zero division when  $\varphi$  is small.

From Eq.(7),

$$\varphi = \frac{\partial e}{\partial \theta} = \frac{K}{K\sigma} y_m \tag{10}$$

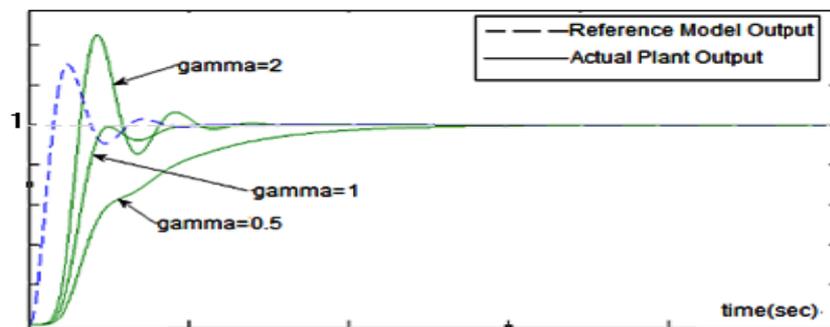
Eq.(9) is also applicable in the conditions when there are more than one adjustable parameters. With the above modifications using normalized algorithm, the adaptation law is referred as Modified MIT rule.

### 5. Simulations and Results

In this paper, the MRAC approach is applied to a second order system with MIT rule and modified MIT rule, and the simulation results are shown below. Fig.(4) shows the response of actual plant and reference model for different values of adaptation gain  $\gamma$ . Table(1) summarizes the dynamic behavior of the system in terms of time domain parameters for various values of  $\gamma$ . It is clear from fig.(4) that,for large values of  $\gamma$  system responses fast with larger overshoots, and for small values of  $\gamma$  system responses slow with small overshoot.In this paper the span of gain  $\gamma$  is chosen from 0.5 to 5 for the given system. Beyond this span the system performance is not satisfactory.

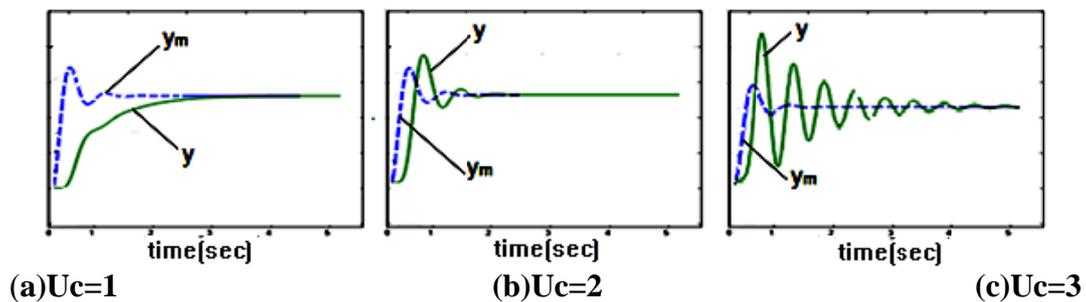
**Table 1:** Comparison between the responses of the system for various values of adaptation gain.

Adaptation	Peak	Time	Peak Overshoot	Settling Time
0.5	0	0	0	21.5
1	0	0	0	8.6
2	4.26	45	45	10.2
3	3.9	74	74	15.6

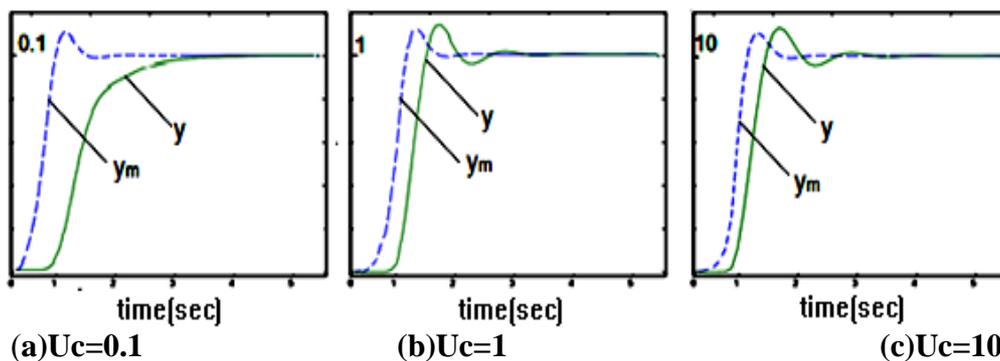


**Figure 4:** Simulation result of MRAC with MIT rule for various values of adaptation gain ( $\gamma$ ).

Fig.(5-6) depict the responses of MIT and modified MIT based controllers when subjected to different amplitudes of reference input. Fig.(5) indicates that the controller designed with MIT rule is very sensitive to the change in amplitude of reference input and may become unstable for larger values of reference input. From fig.(6), we can see that MIT rule with normalized algorithm makes the system almost insensitive to input amplitude changes.



**Figure 5:** Simulation result of MRAC with MIT rule subjected to different amplitudes of input signal  $U_c$ .



**Figure 6:** Simulation result of MRAC with modified MIT rule subjected to different amplitudes of input signal  $U_c$ .

## 6. Conclusions

A detailed discussion on MRAC scheme using MIT rule is done in this paper and the performance evaluation is carried out by means of simulations on SIMULINK. Table (1) compares the results of MIT scheme for different values of adaptation gain. It has been observed from fig.(4) that the response of the system improves with the increment in adaptation gain but beyond a certain limit ( $5 > \gamma > 0.5$ ) the performance of the system becomes very poor.

In this paper, the MIT rule is applied in many different cases. The selection of adaptation gain is very important and depends on the signal levels. The Normalized algorithm, used in this paper, is less sensitive even for very large and very small amplitudes of reference input. Therefore, it is shown in this paper that for suitable values of adaptation gain, the MIT rule with normalization can make the plant to follow the model as accurately as possible.

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